Функции нескольких переменных.

1. Найти частные производные.

1. $z = 2x^2y + 3xy^2 + x^3$	2. $u = xy + y \cdot \sqrt[4]{z} + xz$	3. $z = x^3 + \sqrt{y} - 5xy$
4. $z = x\sin\left(xy^2\right)$	5. $z = \exp\left(-x^2y^2\right)$	6. $z = x\sin\left(x+y\right)$
7. $u = arctg\left(\dfrac{z}{xy}\right)$	8. $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$	9. $z = \ln\left(x^3 + xy + 2y^3\right)$
10. $z = x\cos\left(2xy\right)$	11. $z = \dfrac{x}{\sqrt{4x^2+y^2}}$	12. $z = 3^{\ln(x+2y)}$
13. $z = \sin\dfrac{x}{y}\cos\dfrac{x}{y}$	14. $u = x^3 + y^5 - 4x^2z^2$	15. $u = \ln\left(3z + \sqrt{8x^2+y^2}\right)$
16. $u = \sqrt{5x^2 + y^2x - z^2}$	17. $z = \arccos\left(\dfrac{y}{x}\right)$	18. $z = tg\left(\dfrac{y^2}{x}\right)$
19. $u = \dfrac{\sqrt{x}}{y} + \dfrac{y}{\sqrt{z}} + \dfrac{z}{x}$	20. $z = 7x^3y - xy^7$	21. $z = \ln\sqrt{x^2y + 9y^2}$
22. $z = \exp\left(\sin x \cdot \sqrt[3]{y}\right)$	23. $z = xy\arcsin x^2$	24. $u = y\sqrt{x} + yz + x\sqrt{z}$
25. $z = x \cdot \exp\left(x^2 - y^2\right)$	26. $z = \arcsin\left(\dfrac{x}{y}\right)$	27. $u = \sin\left(x^2yz^2\right)$
28. $z = \cos\left(x^2y^3\right)$	29. $z = xarctg\left(xy^2\right)$	30. $z = 5^{3x^2-y^2}$

2. Для функции $u = u(x,y,z)$ найти градиент и производную по направлению $\vec{l}$ в точке M.

1. $u = 2x^2 + 3xy + zy,\ \vec{l} = \{3,4,0\},\ M(1,-1,1)$	2. $u = x^2 + xy - 2zy,\ \vec{l} = \{1,2,2\},\ M(1,2,1)$
3. $u = 3x^2 - 4xy + 3zy,\ \vec{l} = \{0,3,4\},\ M(-2,1,1)$	4. $u = 2x^2 - 3yz + 2zx,\ \vec{l} = \{2,1,-2\},\ M(-1,1,4)$
5. $u = 2xy + zy - 5xz^2,\ \vec{l} = \{-4,0,3\},\ M(0,1,5)$	6. $u = 3xy^2 - 2x^2 - 5zy,\ \vec{l} = \{2,-2,1\},\ M(1,-1,0)$
7. $u = x^2y^2 + x^2z^2 + z^2y^2,\ \vec{l} = \{1,4,0\},\ M(0,1,-5)$	8. $u = x^2y - z^2 + 2zy^2,\ \vec{l} = \{-1,2,2\},\ M(1,3,-4)$
9. $u = xy + 2xz^2 - zy,\ \vec{l} = \{-3,4,-1\},\ M(2,1,5)$	10. $u = 2x^2y - xz + z^2y,\ \vec{l} = \{-2,4,-4\},\ M(3,1,-6)$
11. $u = x^2 - 5xy^2 + 3zy,\ \vec{l} = \{4,0,3\},\ M(2,2,1)$	12. $u = 4x^2y - 3xy + xz^2,\ \vec{l} = \{-2,2,1\},\ M(3,2,1)$
13. $u = 3xy^2 + 3yz^2 - zy^2,\ \vec{l} = \{1,2,0\},\ M(1,2,1)$	14. $u = x^2y - 4yz + 5z^2x,\ \vec{l} = \{1,-2,2\},\ M(2,1,1)$
15. $u = 5xy - yz^2 - zx^2,\ \vec{l} = \{0,1,2\},\ M(2,2,1)$	16. $u = x^2 + 5xy - zy^2,\ \vec{l} = \{2,-1,2\},\ M(1,1,1)$
17. $u = 3x - 4yx^2 + yz^2,\ \vec{l} = \{2,2,1\},\ M(-3,2,1)$	18. $u = 5x^2 + 3xy^2 - 2yz,\ \vec{l} = \{2,-2,1\},\ M(2,1,1)$
19. $u = 2x - yx^2 + 5zy^2,\ \vec{l} = \{-1,2,0\},\ M(1,3,1)$	20. $u = xy^2 - 2xy + 3zx^2,\ \vec{l} = \{1,2,-1\},\ M(3,1,1)$
21. $u = x^2y + 3yz - xz^2,\ \vec{l} = \{0,-1,2\},\ M(1,1,3)$	22. $u = xz^2 + 2yz^2 - 3zx^2,\ \vec{l} = \{1,2,-2\},\ M(2,1,2)$
23. $u = x^2z - 5y^2z + yz^2,\ \vec{l} = \{-2,0,1\},\ M(3,1,3)$	24. $u = 2xy + yz^2 - 2zy^2,\ \vec{l} = \{1,2,-2\},\ M(2,2,1)$
25. $u = 3xy^2 - 3yz^2 - 3zx^2,\ \vec{l} = \{1,-2,0\},\ M(1,2,2)$	26. $u = 5x^2y - yz^2 + zx^2,\ \vec{l} = \{2,1,-2\},\ M(1,3,2)$
27. $u = 5xz^2 + yx^2 - xz^2,\ \vec{l} = \{0,1,-2\},\ M(1,1,2)$	28. $u = 3xz - 2xy^2 - 2yz,\ \vec{l} = \{2,2,1\},\ M(2,2,1)$
29. $u = 5xy + 3yx^2 + zy^2,\ \vec{l} = \{2,0,-1\},\ M(2,3,1)$	30. $u = 4xy + zx^2 - 3zy^2,\ \vec{l} = \{0,3,-4\},\ M(1,1,3)$

3. Найти уравнения касательной плоскости и нормали в указанной точке.

1. $x^2+y^2+z^2=9,\quad M(2,1,2)$	2. $x^2+2x+y^2-z^2=15,\quad M(2,1,2)$
3. $2x^2-y^2-z^2=16,\quad M(4,0,4)$	4. $x^2+2y^2-3z^2=0,\quad M(1,1,1)$
5. $2x^2+y^2-2z=0,\quad M(-2,2,6)$	6. $x^2+y^2-2y+z^2=15,\quad M(0,1,4)$
7. $x^2+4x+y^2-z^2+2z=33,\quad M(-2,6,1)$	8. $x^2-2y^2-z^2=28,\quad M(6,4,0)$
9. $3x^2+2y^2-z^2=0,\quad M(-1,1,1)$	10. $x^2+2x+y^2+z^2=8,\quad M(-1,0,3)$
11. $x^2+y^2-2z=0,\quad M(-2,2,4)$	12. $x^2+y^2-z^2+2z=5,\quad M(0,2,1)$
13. $3x^2-y^2-z^2=28,\quad M(-4,2,4)$	14. $2x^2+y^2-3z^2=0,\quad M(-1,1,1)$
15. $x^2+2y^2-2z=0,\quad M(-2,1,3)$	16. $x^2+y^2+z^2-2z=3,\quad M(-2,0,1)$
17. $x^2+y^2+2y-z^2=8,\quad M(3,-1,0)$	18. $x^2-y^2-2z^2=18,\quad M(6,0,3)$
19. $2x^2+2y^2-z^2=0,\quad M(-2,2,4)$	20. $x^2+4y^2-6z=0,\quad M(-6,0,6)$
21. $x^2-2x+y^2-2y+z^2=7,\quad M(1,1,-3)$	22. $x^2+y^2-z^2=36,\quad M(4,6,-4)$
23. $x^2-y^2-z^2=11,\quad M(6,-5,0)$	24. $2x^2+2y^2-z^2=0,\quad M(2,-2,4)$
25. $x^2+y^2-z^2=0,\quad M(-1,0,1)$	26. $2x^2+3y^2-4z=0,\quad M(2,-2,5)$
27. $2x^2-y^2-4z=0,\quad M(2,2,1)$	28. $x^2-3y^2-3z^2=21,\quad M(-6,2,1)$
29. $x^2-y^2-2z=0,\quad M(2,4,-6)$	30. $x^2+2y^2+2z^2=4,\quad M(0,1,-1)$

4. Провести исследование функции на экстремум.

1. $z=x^3-y^3-3x^2+3y-24x$	2. $z=2xy^2-2x^3+2y^2+24x$	3. $z=4xy+x^2+y^2-6y$
4. $z=x^3+2xy+2y^2+2x^2-36x$	5. $z=x^3-6xy+3y^2-18x-6y$	6. $z=x^3+2xy-y^2-7x+2y$
7. $z=-4xy+8x^2+4y^2-4y$	8. $z=x^3-6xy+3y^2-9x$	9. $z=x^2y+2x^2-3y^2+4y$
10. $z=y^3-4xy-4x^2-y^2-27y$	11. $z=xy^2-2x^3+2y^2+8x$	12. $z=y^3+6xy+3x^2+6y^2-9y$
13. $z=x^2+6xy-4y^3$	14. $z=y^3+3x^2y-3x^2-12y$	15. $z=4xy-2x^2-y^3+4y$
16. $z=2x^3+12xy-3y^2+18x$	17. $z=x^3-y^3-3x+12y$	18. $z=xy-x^2+3x-y$
19. $z=2xy+x+2y^2+2y$	20. $z=2x^3+2xy+y^2-4x$	21. $z=x+3y+4x^2+3y^2$
22. $z=2x^3-6xy-6x^2+3y^2$	23. $z=x^3-24xy-8y^3$	24. $z=8xy+6x^3+4y^2-5x^2$
25. $z=x^2+2xy+4y^2+6y$	26. $z=x-y-4x^2-2y^2$	27. $z=x^2+2xy+4y^2+4y$
28. $z=y^3+3xy-x^3$	29. $z=2x^3+6xy-y^2+12x$	30. $z=2x-2xy+y^2-4y$

5. Применяя метод градиента, найти экстремум функции на заданном множестве.

1. $f(x,y)=x+y,$ $\begin{cases}x+2y\le 0,\\ x-2y-15\le 0,\\ 2x+y+7\ge 0.\end{cases}$	2. $f(x,y)=x+y+1,$ $\begin{cases}x\le 6,\\ y\le 3,\\ x+2y\ge 0.\end{cases}$	3. $f(x,y)=x-y,$ $\begin{cases}x+2y\ge 0,\\ x-2y+15\ge 0,\\ 2x+y-6\le 0.\end{cases}$

4. $f(x,y)=x-y+1,$ $\begin{cases}2x+y-3\le 0,\\ x-2y-15\le 0,\\ x\ge 0.\end{cases}$	5. $f(x,y)=x-y-2,$ $\begin{cases}2x-y+4\ge 0,\\ x+3y+15\ge 0,\\ x\le 0.\end{cases}$	6. $f(x,y)=2x+y-3,$ $\begin{cases}x\ge -4,\\ y\le 2,\\ x-2y\le 0.\end{cases}$
7. $f(x,y)=2x-y+6,$ $\begin{cases}x\le 8,\\ x+y\le 9,\\ 2x+y\ge 0.\end{cases}$	8. $f(x,y)=x-2y-2,$ $\begin{cases}3x-y\ge 0,\\ x-2y-9\le 0,\\ 2x+y-2\le 0.\end{cases}$	9. $f(x,y)=3x-y+3,$ $\begin{cases}2x-2y\le 7,\\ x+2y+9\ge 0,\\ y\le 0.\end{cases}$
10. $f(x,y)=x-4y-4,$ $\begin{cases}4x+2y\ge -1,\\ x-2y-9\le 0,\\ y\le 0.\end{cases}$	11. $f(x,y)=x+3y-6,$ $\begin{cases}x-2y+9\ge 0,\\ 2x+y+2\ge 0,\\ y\ge 2x.\end{cases}$	12. $f(x,y)=x+2y+3,$ $\begin{cases}x+3y-9\le 0,\\ 2x-y+4\le 0,\\ y\ge 0.\end{cases}$
13. $f(x,y)=x+y,$ $\begin{cases}x+4y\le 0,\\ x-y-6\le 0,\\ 3x+y+9\ge 0.\end{cases}$	14. $f(x,y)=x-y-3,$ $\begin{cases}x\le 4,\\ x+y\le 1,\\ x+3y\ge 0.\end{cases}$	15. $f(x,y)=x-y+5,$ $\begin{cases}x+y\ge 0,\\ x-4y+16\ge 0,\\ 3x+y-12\le 0.\end{cases}$
16. $f(x,y)=2x-y+7,$ $\begin{cases}x+y-8\le 0,\\ x-4y-20\le 0,\\ x\ge 0.\end{cases}$	17. $f(x,y)=-2x+y-1,$ $\begin{cases}x-y+5\ge 0,\\ x+3y+9\ge 0,\\ x\le 0.\end{cases}$	18. $f(x,y)=x+3y-7,$ $\begin{cases}x\ge -3,\\ 3x+y\le 0,\\ x+y+1\ge 0.\end{cases}$
19. $f(x,y)=4x+y+1,$ $\begin{cases}5x+2y\le 6,\\ y\le 8,\\ 2x+y\ge 0.\end{cases}$	20. $f(x,y)=x-3y+1,$ $\begin{cases}5x-y\ge 0,\\ x-2y-4\le 0,\\ 4x+y-16\le 0.\end{cases}$	21. $f(x,y)=-x+y-1,$ $\begin{cases}4x-y\le 8,\\ x+y+1\ge 0,\\ y\le 0.\end{cases}$
22. $f(x,y)=x-2y-2,$ $\begin{cases}x+2y+6\ge 0,\\ x-3y-6\le 0,\\ y\le 0.\end{cases}$	23. $f(x,y)=-2x+y-3,$ $\begin{cases}x-3y+9\ge 0,\\ 4x+y+8\le 0,\\ y\ge 3x.\end{cases}$	24. $f(x,y)=x+2y+3,$ $\begin{cases}x+4y-4\le 0,\\ x-y+6\ge 0,\\ y\ge 1.\end{cases}$
25. $f(x,y)=x+3y-7,$ $\begin{cases}2x-y+5\ge 0,\\ 2x+2y-1\le 0,\\ y+9\ge 0.\end{cases}$	26. $f(x,y)=3x+y-2,$ $\begin{cases}x-2y+4\ge 0,\\ x+2y+4\ge 0,\\ x\le 8.\end{cases}$	27. $f(x,y)=x-5y+1,$ $\begin{cases}2x-y-10\le 0,\\ 2x+y+5\ge 0,\\ y\le 1.\end{cases}$
28. $f(x,y)=3x+2y-2,$ $\begin{cases}x+2y-9\le 0,\\ 4x-y-4\le 0,\\ x+3\ge 0.\end{cases}$	29. $f(x,y)=2x-3y+5,$ $\begin{cases}x-y+6\ge 0,\\ 3x+y+1\le 0,\\ x\le 0.\end{cases}$	30. $f(x,y)=x-3y+4,$ $\begin{cases}x+y-3\le 0,\\ x-4y-1\le 0,\\ x+9\ge 0.\end{cases}$